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The influence of the notches on the natural frequencies

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Abstract

The simplification of the FEM model (the missing of a notch or a crack) generally means the modification of the natural frequencies. An analysis of these changes is matter of this paper. The direct and semicircle notches were made on the testing specimens (by cut depth in the specimens) and the naturals frequencies were measured and computed. The results are compared.

In the paper is discussed the choice of the FEM element for FEM model and the comparison with experiment is made too.

The influence of the crack on the natural frequencies is discussed in the case of specimen for high cycle fatigue test. The changes of the first three frequencies are compared for specimens with and without crack.

Keywords: natural frequency, FRF, notch, crack, FEM

1. Introduction

The usually questions in finite element (FE) computing are related to the memory requirements and time consumption. From this point of view our aim is the simplification of the FE model for example missing of some details. The simplification implies error estimation after this action and this estimation cannot be trivial. This task is affected by the choice of the type of element too. In the case we have not sufficient knowledge about this matter or in the first analysis stage we often use the implicit values (for example tetrahedron for solids).

If the results are represented by “the perfectly colored pictures” then it is easy to believe the results are correct (without the detail analysis of results, inputs parameters and applied methods).

The FE model is impossible to reduce in the same special cases because the neglect of the local effect leads to the serious change of natural frequency. This information can be using to the detection possible failure of the object (for example the crack in the structure). On the relationship between natural frequency and defect can be established decision rules for condition monitoring.

The idea to forewarn a final failure on the base of changes of modal data was used by [6] for spot welded joints and for composite materials, general algorithms was presented by [13] to apply modal analysis for damage detection, etc. The relationships between dynamical (modal) properties and optimization of structured were solved in [3], [4], [8], [10], [11] and [14].

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2. Testing specimens and solution processing

2.1. The testing specimens

We are dealt with three types of specimens. The first two types (A and B) are steel beams (STN 411737) 490 mm in long with rectangular cross-section (20 x 5 mm, $s \times h$) with and without the notches in the middle of the beams. The first type A with notch A (a width of 0.35 mm) is depicted in fig 1. The second type B with notch B (a width of 0.8 mm) is depicted in fig. 2.

The notches were made with the different depth, for type A or B a depth of s/i or (h/i), fig. 1 and fig. 2, where:

$$i \in \left\{4, 2, \frac{4}{3}\right\}. \tag{1}$$

The third type C is specimen for high frequency fatigue test [1] and [7], (from AISI 316L) without the crack and with crack (a depth of 40% s in the reduce part).

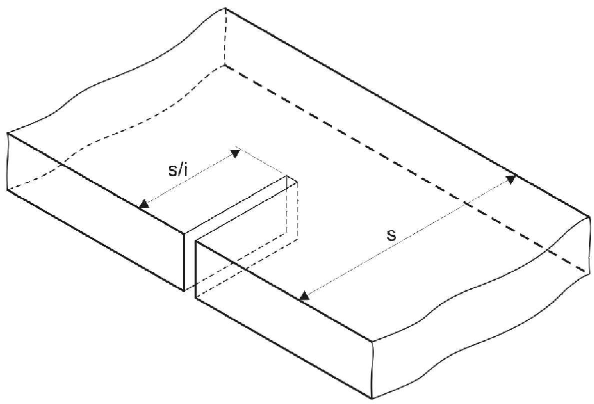

Fig. 1. The notch A in the middle of the length beams, with the different depth s/i.

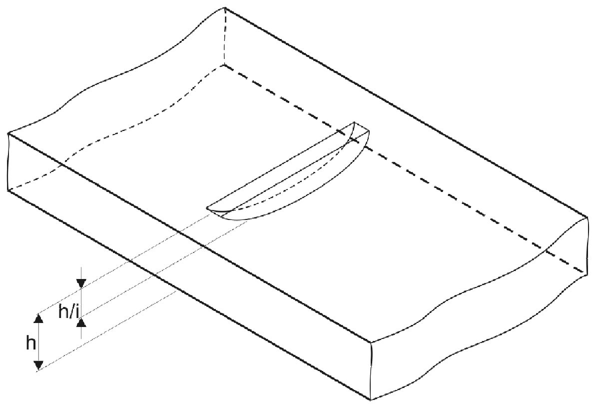

Fig. 2. The notch B in the middle of the length beams, with the different depth h/i.

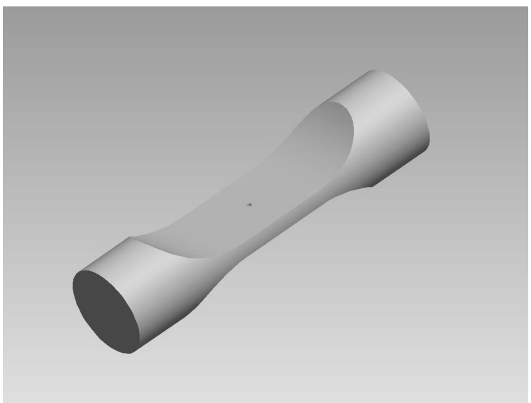

Fig. 3. The specimen C for high frequency fatigue test.

2.2. The experiments

The natural frequencies on the base of frequency response function (FRF) [9] for free-free vibration are measured for all specimens without and with the notch or the crack. These frequencies are compared for different depth of notches. The change of the frequencies is a measure of error in the case of missing of the notch or the crack.

The PC card NI4472 (the product of National Instrument, NI), with accelerometer ACC104A (Omega), impulse hammer IH101-500 (Omega) and LabView 8.0 with Sound and Vibration Toolkit (NI) were used for all measurement.

2.3. Finite element analysis

The commercial software products ADINA and ANSYS were used. The 4-node tetra, 8-node brick and 10-node tetra elements were preferred. The other elements were used too but in this paper the results was not published.

The computations were made for specimens A and B with and without notches and for specimen C without the crack.

2.4. The analytical solution for specimen without notch

A differential equation, which is governed the elastic curve $w(x,t)$ characterizing the shape of the deformed beam in the place x and in the time t [2], [5] and [12].

$$\frac{\partial^4 w(x,t)}{\partial x^4} + \frac{\rho S}{EJ}\frac{\partial^2 w(x,t)}{\partial t^2} = 0\,, \tag{2}$$

where ρ is density, S is cross-section area, E is modulus of elasticity and J is moment of inertia of the area S. The naturals frequencies f_n for bending vibration is explained (on the base of roots of characteristic equation for (2)):

$$f_n = \frac{\lambda_n^2}{2\pi\, l^2}\sqrt{\frac{E\,J}{\rho\,S}}\,, \qquad \lambda_n = (n+0.5)\pi\,, \qquad n \in \{1,2,3,\ldots\}, \tag{3}$$

where l is length of the beam.

The first four natural frequencies for beam without notch are 108.84 Hz; 302.33 Hz; 435.36 Hz and 592.57 Hz.

We reduced application only first four natural frequencies and we did not concern on torsion and longitudinal vibration in this paper.

3. The results of measurement and computing

3.1. The results for notches

Frequency response functions (RTF's) are depicted for specimens A and B. In the fig. 4 and fig. 5 are depicted FRF's for specimen A with and without notches. In the fig. 5 and fig. 6 are depicted FRF's for specimen B with and without notches too. The 3rd natural frequency (NF) is depicted in the separate figure because we used 1-axis accelerometer and the plane of the motion of 3rd modal shape is perpendicular to the planes of 1st, 2nd and 4th modal shapes.

In the fig. 4, fig. 6 and fig. 7 there are depicted FRF's only for specimen without notch and for maximal depth of notch. The other peeks of FRF's are placed between these boundary peeks and all these curves are not shown in fig. 4 to fig. 7.

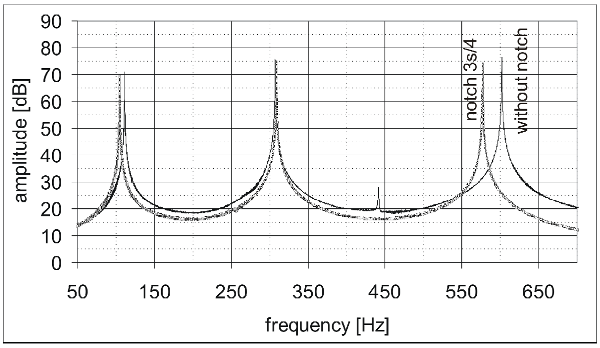

Fig. 4. The FRF's of specimen A for 1st, 2nd and 4th natural frequency.

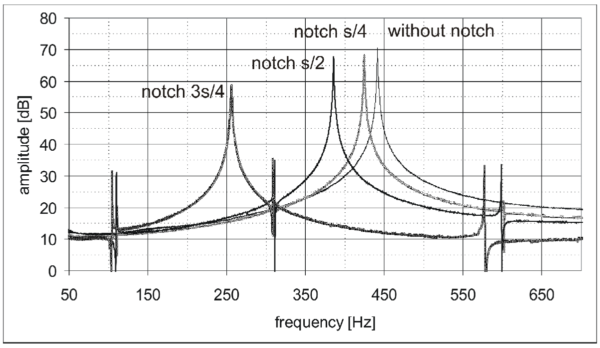

Fig. 5. The FRF's of specimen A for 3rd natural frequency.

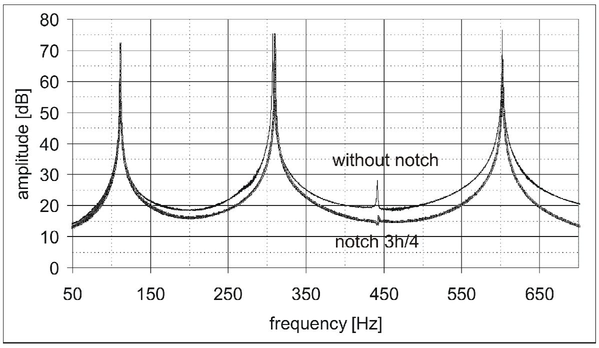

Fig. 6. The FRF's of specimen A for 1st, 2nd and 4th natural frequency.

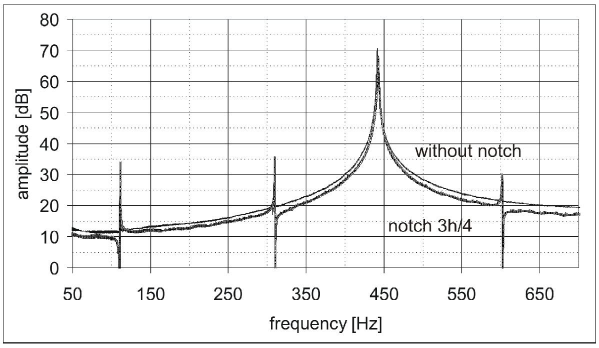

Fig. 7. The FRF's of specimen B for 3rd natural frequency.

The first four natural frequencies from experiment, analytical solution and finite element analysis (FEA) by using ADINA and ANSYS are presented in the tab. 1. The element size of 1 mm is used in these computations. The variation of the natural frequencies values between 8-node and 10-node elements was negligible.

No.	Natural frequency [Hz]						
	Analytical solution	Experiment	ADINA		ANSYS		
			4-node elements	8-node elements	4-node elements	8-node elements	10-node elements
1.	108,8	111,2	118,2	110,8	115,3	109,6	109,6
2.	302,3	307,1	325,6	305,3	317,6	302,0	302,0
3.	435,4	441,3	438,2	436,3	437,1	436,0	436,0
4.	592,6	602,1	637,8	598,1	622,2	591,7	591,7

Tab. 1. The values of natural frequencies for beam without notch.

In the tab. 2 are presented variations of the values from tab. 1. In the tab. 3 and tab. 4 are presented the values and variations of natural frequencies of specimen A. The 3rd natural frequency is changed substantial. This fact is presented on the fig. 5. ANSYS is used for computation of natural frequencies in the tab. 3 and all next tables too.

No.	Variation of FEA and experimental NF from analytical solution [%] Variation of FEA and analytical solution NF from experiment [%]					
	Analytical solution	Experiment	ADINA		ANSYS	
			4-node	8-node	4-node	8-node
1.	0.0 -2.1	2.2 0.0	6.3 8.6	1.8 -0.4	5.9 3.7	0.7 -1.4
2.	0.0 -1.5	1.6 0.0	6.0 7.7	1.0 -0.6	5.1 3.4	-0.1 -1.6
3.	0.0 -1.3	1.4 0.0	-0.7 0.7	0.2 -1.1	0.4 -1.0	0.2 -1.2
4.	0.0 -1.6	1.6 0.0	6.0 7.6	0.9 -0.7	5.0 3.3	-0.2 -1.7

Tab. 2. The comparison of NF and the variations for beam without notch for different element.

No.	Natural frequency [Hz]			
	Without notch	s/4	s/2	3s/4
	FEA Experiment	FEA Experiment	FEA Experiment	FEA Experiment
1.	110,8 111,2	111,3 110,7	109,4 110,0	104,8 104.3
2.	305,3 307,1	308,5 308,5	309,0 311,0	308,2 308,4
3.	436,3 441,3	427,1 424,0	379,0 385,6	257,3 255,7
4.	598,1 602,1	601,3 601,4	594,0 598,4	576,5 576,8

Tab. 3. The values of natural frequencies for beam with notch A.

No.	Variation of FEA with notch from FEA without notch [%] Variation of FEA from experiment [%] Variation of experiment with notch from experiment without notch[%]		
	Notch A		
	s/4	s/2	3s/4
1.	0.5 0.5 -0.4	-1.3 -0.5 -1.1	-5.4 0.5 -6.2
2.	1.1 0.0 0.5	1.2 -0.6 1.3	1.0 -0.1 0.4
3.	-2.1 0.7 -3.9	-13.2 -1.7 -12.6	-41.0 0.6 -42.1
4.	0.6 0.0 -0.1	-0.7 -0.7 -0.6	-3.6 -0.1 -4.2

Tab. 4. The comparison of natural frequencies (the variations) for beam with notch A.

The variations of natural frequencies for specimen B are negligible and the similar tables for this specimen are not presented. This fact is presented in the fig. 6 and fig. 7.

The first four modal shapes are depicted in fig. 8 to fig. 11. These figures are useful to understanding that the 3rd frequency is changed in a great measure. This modal shape is performed in the plane XY and the notch A is opened.

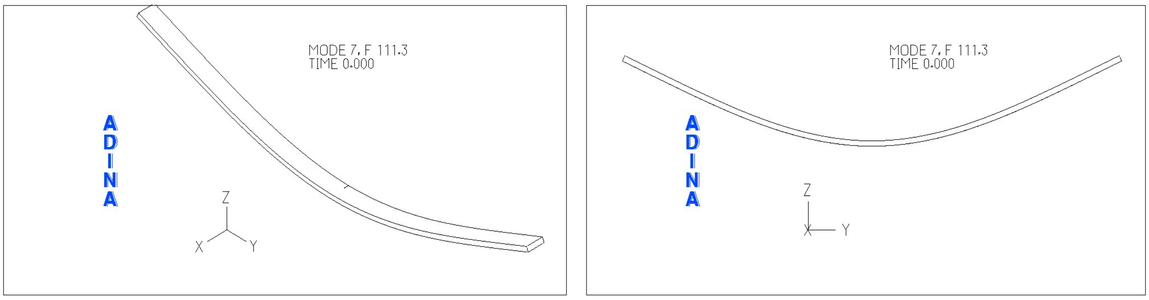

Fig. 8. 1st modal shape in the plane YZ.

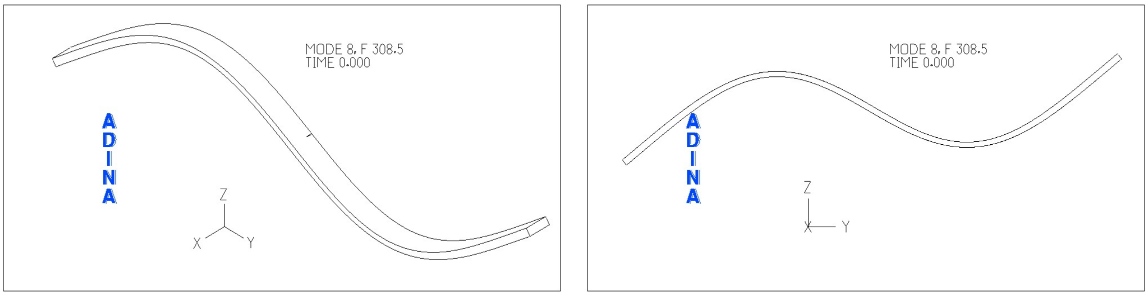

Fig. 9. 2nd modal shape in the plane YZ.

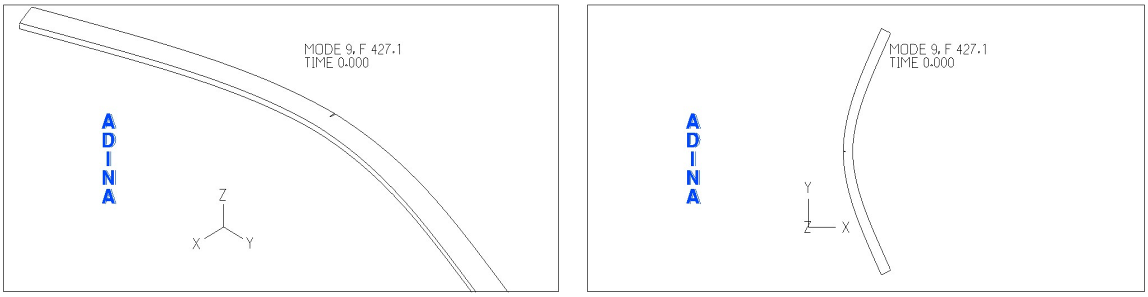

Fig. 10. 3rd modal shape in the plane XY.

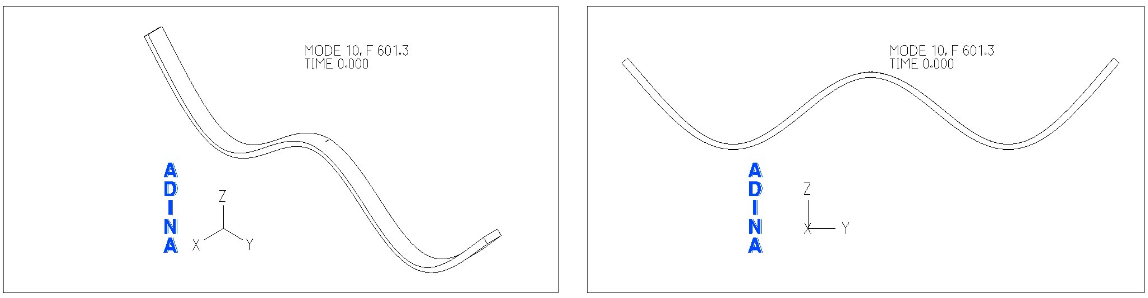

Fig. 11. 4th modal shape in the plane YZ.

3.2. The results for crack

The FRF's (amplitude and phase) for specimen C with and without the crack are shown in the fig. 9 and fig. 10. The natural frequencies for specimen with crack are shifted in links.

The natural frequencies from experiment and FEA are presented in the tab. 5 with variation of FEA from experiment. In the tab. 5 there are presented results for type of element, size of element and mesh method too.

The natural frequencies and their variations are presented in the tab. 6 with and without crack. The frequencies of specimen with crack are substantially decreased.

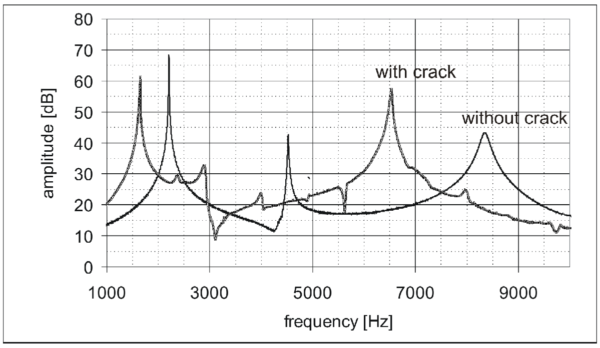

Fig. 9. The amplitude of FRF's of specimen C for 1st, 2nd and 3rd natural frequency.

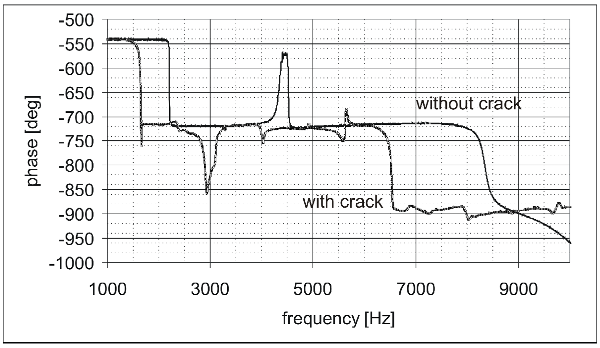

Fig. 10. The phase of FRF's of specimen C for 1st, 2nd and 3rd natural frequency.

No	Experiment [Hz]	FEA natural frequency [Hz] Variation of FEA from experiment [%]				
	2203	4-node, Free mesh	4-node 1 mm	4-node 0.5 mm	10-node Free mesh	10-node 1 mm
1	2203	4658 111.4	2504 13.7	2360 7.1	2342 6.3	2297 4.3
2	4525	10217 125.8	5210 15.1	4877 7.8	4908 8.5	4731 4.6
3	8349	11893 42.4	9098 9.0	8649 3.6	8842 5.9	8447 1.2

Tab. 5. Comparison of natural frequencies and variations for specimen C without crack from the experiment and FEA.

No.	Experimental natural frequency [Hz]		Variation NF for specimen without crack from specimen with crack [%]
	Without crack	With crack	
1	2203	1644	-34.0
2	4529	2859	-58.3
3	8349	6520	-28.1

Tab. 6. Comparison of natural frequencies and variations for specimen C without and with crack from the experiment.

7. Conclusion

The paper deals about measurement ant computation of natural frequencies for three types of specimens. We were concentrated on comparison of the natural frequencies for specimens with and without notches and with and without crack. The computed natural frequencies by using different types and sizes of elements are compared with experimental results.

We should like to point out that the uncritical using default values of element types and mesh method can be unsuitable. The using of default values can be generally useful in the first steps of the analysis.

The influence of notch B on the natural frequencies is negligible and in the case A is influence various for different natural frequencies. The 3rd modal shape is realized in the plane of opening notch. The influence of geometric parameters of notch is important in this plane of 3rd modal shape.

The variance between natural frequencies of specimen without and with crack is significant. We do not make a conclusion about miss out a crack from FE model because we have not an appropriate FE model for crack.

The presence of natural frequency in spectrum is generally presented as a serious problem of machine in the condition monitoring publications. The change of this frequency can identify a growth of crack.

Acknowledgements

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